

$U(3)_L \times U(3)_R$ Chiral Theory of Mesons

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Abstract

The chiral theory of mesons of two flavors have been extended to mesons containing strange flavor. Two new mass relations between vector and axial-vector mesons have been obtained. In chiral limit, the physical processes of normal parity and abnormal parity have been studied. Due to the universality of the coupling of this theory many interesting results have been obtained. In the chiral limit theoretical results are in reasonable agreement with data.

Chiral symmetry is one of the most important features revealed from quantum chromodynamics (QCD). Based on chiral symmetry and minimum coupling, a meson theory of two flavors has been proposed[1] and the theoretical results are in good agreement with the phenomenology of pseudoscalar, vector, and axial-vector mesons made of u and d quarks. In this paper we generalize the study of ref.[1] to K , η , η' , $K^*(892)$, ϕ , $K_1(1400)$, and $f_1(1510)$ mesons containing the third flavor-strange quark. The paper is organized as follows. 1) the formalism of the theory; 2) new mass relations between vector and axial-vector mesons; 3) Vector meson dominance(VMD) and kion-form factors; 4) Decays of τ lepton; 5)Decays of ϕ , K^* , $K_1(1400)$, $f_1(1510)$ and η' mesons; 6) Decays of $K^*(892) \rightarrow K\pi\pi$; 7) Electromagnetic decays of mesons; 8) Summary of the results; 9) Conclusion.

The formalism of $U(3)_L \times U(3)_R$ chiral theory of mesons

Using $U(3)_L \times U(3)_R$ chiral symmetry and the minimum coupling principle, the lagrangian of quarks of three flavors and other fields has been constructed as

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(x)(i\not{\partial} + \not{p} + e_0 Q \not{A} + \not{p}\gamma_5 - mu(x))\psi(x) + \frac{1}{2}m_1^2(\rho_i^\mu \rho_{\mu i} + \omega^\mu \omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu) \\ & + \frac{1}{2}m_2^2(K_\mu^{*a} \bar{K}^{*a\mu} + K_1^\mu K_{1\mu}) + \frac{1}{2}m_3^2(\phi_\mu \phi^\mu + f_s^\mu f_{s\mu}) \\ & + \bar{\psi}(x)_L \not{W} \psi(x)_L + \mathcal{L}_{EM} + \mathcal{L}_W + \mathcal{L}_{lepton} \end{aligned} \quad (1)$$

where $a_\mu = \tau_i a_\mu^i + \lambda_a K_{1\mu}^a + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)f_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)f_{s\mu}$ ($i = 1, 2, 3$ and $a = 4, 5, 6, 7$),

$v_\mu = \tau_i \rho_\mu^i + \lambda_a K_\mu^* + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)\omega_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)\phi_\mu$, A_μ is photon field, Q is the electric charge operator of u , d , and s quarks, W_μ^i is W boson, and $u = \exp i\{\gamma_5(\tau_i \pi_i + \lambda_a K^a + \eta + \eta')\}$, m is a parameter. In eq.(1) u can be written as

$$u = \frac{1}{2}(1 + \gamma_5)U + \frac{1}{2}(1 - \gamma_5)U^\dagger, \quad (2)$$

where $U = \exp i(\tau_i \pi_i + \lambda_a K^a + \eta + \eta')$. In eq.(1) ψ are u , d , and s quark fields which carry colors and other quantum numbers of quark. All other fields are colorless. The physical fields related to a_μ and v_μ will be defined below. As mentioned in ref.[1], in QCD the boson fields v_μ , a_μ , and pseudoscalars are not fundamental fields and they should be bound state solutions of QCD . Therefore, in eq.(1) there are no kinetic terms for those fields and the kinetic terms will be generated from quark loops(see below). As a matter of fact, the relationship between boson fields and quark fields can be found from lagrangian(1). Taking ρ_μ^i and a_μ^i fields as examples. Using the least action principle

$$\frac{\delta \mathcal{L}}{\delta \rho_\mu^i} = 0, \quad \frac{\delta \mathcal{L}}{\delta a_\mu^i} = 0,$$

we obtain following relationships

$$\rho_\mu^i = -\frac{1}{m_1^2} \bar{\psi} \tau_i \gamma_\mu \psi, \quad a_\mu^i = -\frac{1}{m_1^2} \bar{\psi} \tau_i \gamma_\mu \gamma_5 \psi.$$

Substituting these relations into eq.(1), except the term $-m\bar{\psi}u\psi$ the hadronic part of the lagrangian(1) becomes Nambu-Jona-Lasinio model[2]. The introduction of the pseudoscalar

fields into lagrangian(1) is based on nonlinear σ model. u of eq.(1) is a series of pseudoscalar fields. In principle, the relationships between pseudoscalar fields and quark fields should be found by the least action principle, but they are not as simple as the relations shown above. This is the difference between present theory and Nambu-Jona-Lasinio model.

It is the same with ref.[1] that using method of path integral to integrating out the quark fields, the effective lagrangian of mesons are obtained. To the fourth order in covariant derivatives in Minkofsky space, the real part of the effective lagrangian describing the physical processes of normal parity takes following form

$$\begin{aligned}
\mathcal{L}_{RE} = & \frac{N_c}{(4\pi)^2} m^2 \frac{D}{4} \Gamma(2 - \frac{D}{2}) Tr D_\mu U D^\mu U^\dagger \\
& - \frac{1}{3} \frac{N_c}{(4\pi)^2} \frac{D}{4} \Gamma(2 - \frac{D}{2}) Tr \{v_{\mu\nu} v^{\mu\nu} + a_{\mu\nu} a^{\mu\nu}\} \\
& + \frac{i}{2} \frac{N_c}{(4\pi)^2} Tr \{D_\mu U D_\nu U^\dagger + D_\mu U^\dagger D_\nu U\} v^{\nu\mu} \\
& + \frac{i}{2} \frac{N_c}{(4\pi)^2} Tr \{D_\mu U^\dagger D_\nu U - D_\mu U D_\nu U^\dagger\} a^{\nu\mu} \\
& + \frac{N_c}{6(4\pi)^2} Tr D_\mu D_\nu U D^\mu D^\nu U^\dagger \\
& - \frac{N_c}{12(4\pi)^2} Tr \{D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger + D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U - D_\mu U D_\nu U^\dagger D^\mu U D^\nu U^\dagger\} \\
& + \frac{1}{2} m_1^2 (\rho_i^\mu \rho_{\mu i} + \omega^\mu \omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu) \\
& + \frac{1}{2} m_2^2 (K_\mu^{*a} \bar{K}^{*a\mu} + K_1^\mu K_{1\mu}) + \frac{1}{2} m_3^2 (\phi_\mu \phi^\mu + f_s^\mu f_{s\mu}),
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
D_\mu U &= \partial_\mu U - i[v_\mu, U] + i\{a_\mu, U\}, \\
D_\mu U^\dagger &= \partial_\mu U^\dagger - i[v_\mu, U^\dagger] - i\{a_\mu, U^\dagger\}, \\
v_{\mu\nu} &= \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu] - i[a_\mu, a_\nu], \\
a_{\mu\nu} &= \partial_\mu a_\nu - \partial_\nu a_\mu - i[a_\mu, v_\nu] - i[v_\mu, a_\nu], \\
D_\nu D_\mu U &= \partial_\nu(D_\mu U) - i[v_\nu, D_\mu U] + i\{a_\nu, D_\mu U\}, \\
D_\nu D_\mu U^\dagger &= \partial_\nu(D_\mu U^\dagger) - i[v_\nu, D_\mu U^\dagger] - i\{a_\nu, D_\mu U^\dagger\}.
\end{aligned}$$

Following ref.[1], the effective lagrangian describing the physical processes with abnormal party will be evaluated in terms of the quark propergators.

In this paper except the kion form factor $f_-(q^2)$, all studies have been done in the chiral limit. In chiral limit, the following definitions[1] are held in this paper

$$\frac{F^2}{16} = \frac{N_c}{(4\pi)^2} m^2 \frac{D}{4} \Gamma(2 - \frac{D}{2}), \quad (4)$$

$$g^2 = \frac{8}{3} \frac{N_c}{(4\pi)^2} \frac{D}{4} \Gamma(2 - \frac{D}{2}) = \frac{1}{6} \frac{F^2}{m^2}. \quad (5)$$

According to the arguments of ref.[1], the physical meson fields have been defined as

$$\begin{aligned}
\rho &\rightarrow \frac{1}{g}\rho, \quad K^* \rightarrow \frac{1}{g}K^*, \quad \omega \rightarrow \frac{1}{g}\omega, \quad \phi \rightarrow \frac{\sqrt{2}}{g}\phi, \\
a_\mu^i &\rightarrow \frac{1}{g(1 - \frac{1}{2\pi^2 g^2})^{\frac{1}{2}}} a_\mu^i - \frac{c}{g} \partial_\mu \pi^i, \quad f_\mu \rightarrow \frac{1}{g(1 - \frac{1}{2\pi^2 g^2})^{\frac{1}{2}}} f_\mu - \frac{c}{g} \partial_\mu \eta_0,
\end{aligned}$$

$$\begin{aligned}
K_{1\mu} &\rightarrow \frac{1}{g(1 - \frac{1}{2\pi^2 g^2})^{\frac{1}{2}}} K_{1\mu} - \frac{c}{g} \partial_\mu K, & f_{s\mu} &\rightarrow \frac{\sqrt{2}}{g(1 - \frac{1}{2\pi^2 g^2})^{\frac{1}{2}}} f_{s\mu} - \frac{c}{g} \partial_\mu \eta_s, \\
\pi &\rightarrow \frac{2}{f_\pi} \pi, & K &\rightarrow \frac{2}{f_K} K, & \eta &\rightarrow \frac{2}{f_\eta} \eta, & \eta' &\rightarrow \frac{2}{f_{\eta'}} \eta',
\end{aligned} \tag{6}$$

where $\eta_0 = (\frac{1}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta)\eta + (\frac{1}{\sqrt{3}}\sin\theta + \sqrt{\frac{2}{3}}\cos\theta)\eta'$ and $\eta_s = (-\frac{2}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta)\eta + (-\frac{2}{\sqrt{3}}\sin\theta + \sqrt{\frac{2}{3}}\cos\theta)\eta'$, θ is the mixing angle of η and η' . In chiral limit we take $f_\pi = f_K = f_\eta = f_{\eta'}$. In chiral limit following two equations of ref.[1] are held in the case of three flavors

$$c = \frac{f_\pi^2}{2gm_\rho^2}, \tag{7}$$

$$\frac{F^2}{f_\pi^2} (1 - \frac{2c}{g}) = 1. \tag{8}$$

Following ref.[1] we have

$$g = 0.35. \tag{9}$$

Use the substitutions(6), the physical masses of vector mesons are defined as

$$m_\rho^2 = m_\omega^2 = \frac{1}{g^2} m_1^2, \quad m_{K^*}^2 = \frac{1}{g^2} m_2^2, \quad m_\phi^2 = \frac{2}{g^2} m_3^2. \tag{10}$$

New mass formulas of vector mesons and its chiral partners

In ref.[1] two mass relations of a_1 ρ and $f_1(1285)$ ω have been obtained

$$\begin{aligned}
(1 - \frac{1}{2\pi^2 g^2}) m_a^2 &= \frac{F^2}{g^2} + m_\rho^2, \\
(1 - \frac{1}{2\pi^2 g^2}) m_f^2 &= \frac{F^2}{g^2} + m_\omega^2.
\end{aligned} \tag{11}$$

By the same reasons obtaining eqs.(11), we obtain other two mass formulas

$$\begin{aligned} (1 - \frac{1}{2\pi^2 g^2})m_{K_1}^2 &= \frac{F^2}{g^2} + m_{K^*}^2, \\ (1 - \frac{1}{2\pi^2 g^2})m_{f_1(1510)}^2 &= \frac{F^2}{g^2} + m_\phi^2. \end{aligned} \quad (12)$$

If input m_a , m_ρ , and f_π , we obtain

$$m_{f_1(1285)} = 1.27 GeV, \quad m_{K_1(1400)} = 1.38 GeV, \quad m_{f_1(1510)} = 1.51 GeV. \quad (13)$$

The deviations from data are about 1%. In Table I, the masses of these mesons are obtained by taking $g = 0.35$.

Weinberg's first sum rule[3] is

$$\frac{g_\rho^2}{m_\rho^2} - \frac{g_a^2}{m_a^2} = \frac{1}{4}f_\pi^2, \quad (14)$$

where g_a and g_ρ are defined in ref.[1]. In order to compare with this sum rule, the four mass formulas(11,12) can be rewritten as

$$\begin{aligned} \frac{m_a^2}{g_a^2} - \frac{m_\rho^2}{g_\rho^2} &= \frac{f_\pi^2 m_\rho^4}{g_\rho^2(4g_\rho^2 - f_\pi^2 m_\rho^2)}, \\ \frac{m_{f_1(1285)}^2}{g_a^2} - \frac{m_\omega^2}{g_\rho^2} &= \frac{f_\pi^2 m_\rho^4}{g_\rho^2(4g_\rho^2 - f_\pi^2 m_\rho^2)}, \\ \frac{m_{K_1(1400)}^2}{g_a^2} - \frac{m_{K^*}^2}{g_\rho^2} &= \frac{f_\pi^2 m_\rho^4}{g_\rho^2(4g_\rho^2 - f_\pi^2 m_\rho^2)}, \\ \frac{m_{f_1(1510)}^2}{g_a^2} - \frac{m_\phi^2}{g_\rho^2} &= \frac{f_\pi^2 m_\rho^4}{g_\rho^2(4g_\rho^2 - f_\pi^2 m_\rho^2)}. \end{aligned} \quad (15)$$

Vector meson dominance(VMD) and kion form factors

Vector meson dominance(VMD) has been revealed from this theory[1]. Besides the ρ and ω dominance

$$\begin{aligned} \frac{e}{f_\rho} \left\{ -\frac{1}{2} F^{\mu\nu} (\partial_\mu \rho_\nu^0 - \partial_\nu \rho_\mu^0) + A^\mu j_\mu^0 \right\}, \\ \frac{e}{f_\omega} \left\{ -\frac{1}{2} F^{\mu\nu} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu) + A^\mu j_\mu^\omega \right\}, \\ \frac{1}{f_\rho} = \frac{1}{2} g, \quad \frac{1}{f_\omega} = \frac{1}{6} g, \end{aligned} \quad (16)$$

the ϕ dominance has been derived from eq.(1)

$$\frac{e}{f_\phi} \left\{ -\frac{1}{2} F^{\mu\nu} (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu) + A^\mu j_\mu^\phi \right\}, f_\phi = -\frac{1}{3\sqrt{2}} g, \quad (17)$$

where j_μ^ϕ is the current that ϕ meson couples to.

The electric kion form factor can be studied by VMD. The effective lagrangian of $KK\gamma$ consists of two parts: kions couple to photon directly and kions couple to vector mesons then the vector mesons couple to photon. In chiral limit, the couplings between kions and ρ , ω , and ϕ mesons can be found from eq.(3)

$$\begin{aligned} \mathcal{L}_{K\bar{K}v} = & -\frac{2\sqrt{2}i}{g} \phi_\mu (K^+ \partial_\mu K^- + K^0 \partial_\mu \bar{K}^0), \\ & + \frac{2i}{g} \omega_\mu (K^+ \partial_\mu K^- + K^0 \partial_\mu \bar{K}^0), \\ & + \frac{2i}{g} \rho_\mu (K^+ \partial_\mu K^- - K^0 \partial_\mu \bar{K}^0). \end{aligned} \quad (18)$$

In eq.(18), the eq.(8) has been used. Using the substitutions

$$\rho_\mu \rightarrow \frac{e}{f_\rho} A_\mu, \quad \omega_\mu \rightarrow \frac{e}{f_\omega} A_\mu, \quad \phi_\mu \rightarrow \frac{e}{f_\phi} A_\mu, \quad (19)$$

in eqs.(18) the direct couplings of $KK\gamma$ can be obtained. Using the couplings $-\frac{1}{2}\frac{e}{f_v}F_{\mu\nu}(\partial_\mu v_\nu - \partial_\nu v_\mu)$ ($v = \rho, \omega, \phi$) of eqs.(16,17) and eqs.(18), the indirect couplings of $KK\gamma$ can be obtained. Adding these two couplings together, the electric form factor of charged kion has been obtained

$$F_{K^+}(q^2) = \frac{1}{3} \frac{m_\phi^2}{m_\phi^2 - q^2} + \frac{1}{6} \frac{m_\omega^2}{m_\omega^2 - q^2} + \frac{1}{2} \frac{m_\rho^2}{m_\rho^2 - q^2}. \quad (20)$$

Because of eq.(8) this form factor is normalized to be one at $q^2 = 0$ and the radius is determined to be

$$\langle r^2 \rangle = \frac{2}{m_\phi^2} + \frac{1}{m_\omega^2} + \frac{3}{m_\rho^2} = 0.33 fm^2. \quad (21)$$

The theoretical result agrees with data[4](see Table I). In the same way, the electric form factor of neutral kion has been obtained

$$F_{K^0}(q^2) = \frac{1}{3} \frac{m_\phi^2}{m_\phi^2 - q^2} + \frac{1}{6} \frac{m_\omega^2}{m_\omega^2 - q^2} - \frac{1}{2} \frac{m_\rho^2}{m_\rho^2 - q^2},$$

$$\frac{\partial F_{K^0}(q^2)}{\partial q^2} \Big|_{q^2=0} = \frac{1}{3m_\phi^2} + \frac{1}{6m_\omega^2} - \frac{1}{2m_\rho^2} = -0.25 GeV^{-2}. \quad (22)$$

The VMD can be applied to study the form factors of $K \rightarrow \pi l \nu$. Let's study $K^+ \rightarrow \pi^0 l \nu$ first. The vertex of $K^*(892)K\pi^0$ has normal parity and can be derived from eq.(3)

$$\mathcal{L}_{K^*K\pi^0} = \frac{i}{g} \{ (\pi^0 \partial^\mu K^- - \partial_\mu \pi^0 K^-) K^{+\mu} + (\pi^0 \partial_\mu K^+ - \partial_\mu \pi^0 K^+) K^{-\mu} \}$$

$$\begin{aligned}
& + \frac{4i}{gf_\pi^2} \left\{ c^2 - \frac{2N_c}{(4\pi)^2} \left(1 - \frac{2c}{g}\right)^2 \right\} \partial^\nu \pi^0 \{ (\partial_\mu K_\nu^- - \partial_\nu K_\mu^-) \partial^\mu K^- \} \\
& - \frac{i}{8\pi^2 g} \left(1 - \frac{2c}{g}\right)^2 \{ (\partial_\nu \pi^0 \partial^{\mu\nu} K^- - \partial_\nu K^- \partial_{\mu\nu} \pi^0) K^{+\mu} + \partial_\nu \pi^0 \partial^{\mu\nu} K^+ - \partial^{\mu\nu} \pi^0 \partial_\nu K^+ \} K_\mu^- \} \quad (23)
\end{aligned}$$

In the decays of $K \rightarrow \pi l \nu$, there are direct coupling $K\pi W$ and indirect couplings $K\pi K^*$ and K^*W . Using the substitution obtained from eq.(1)

$$K_\mu \rightarrow \frac{g_w}{4} g W_\mu \sin \theta_c \quad (24)$$

in eq.(23), the direct coupling can be obtained. It is similar to VMD, the coupling between K^* and W-boson has been obtained from eq.(3)

$$\mathcal{L}_i = -\frac{1}{4} g g_w \{ (\partial_\mu K_\nu^+ - \partial_\nu K_\mu^+) \partial^\mu W^{-\nu} + (\partial_\mu K_\nu^- - \partial_\nu K_\mu^-) \partial^\mu W^{+\nu} \} \quad (25)$$

From eqs.(23,24,25) the indirect coupling has been obtained. Adding the direct and indirect couplings together, the two form factors of $K^+ \rightarrow \pi^0 l \nu$ have been obtained

$$\begin{aligned}
f_+(q^2) &= \frac{1}{\sqrt{2}} \frac{m_{K^*}^2}{m_{K^*}^2 - q^2}, \quad f_-(q^2) = -\frac{1}{\sqrt{2}} \frac{1}{m_{K^*}^2 - q^2} (m_{K^+}^2 - m_{\pi^0}^2), \\
\lambda_+ &= -\lambda_- = \frac{m_{\pi^0}^2}{m_{K^*}^2} = 0.0239, \\
\xi &= \frac{f_-}{f_+} = -\frac{m_{K^+}^2 - m_{\pi^0}^2}{m_{K^+}^2} = -0.284. \quad (26)
\end{aligned}$$

For K_{l3}^0 we have obtained following quantities in the same way with $K^+ \rightarrow \pi^0 l \nu$

$$f_+(q^2) = \frac{1}{\sqrt{2}} \frac{m_{K^*}^2}{m_{K^*}^2 - q^2}, \quad f_-(q^2) = -\frac{1}{\sqrt{2}} \frac{1}{m_{K^*}^2 - q^2} (m_{K^0}^2 - m_{\pi^+}^2),$$

$$\begin{aligned}\lambda_+ &= -\lambda_- = \frac{m_{\pi^+}^2}{m_{K^*}^2} = 0.0245, \\ \xi &= \frac{f_-}{f_+} = -\frac{m_{K^0}^2 - m_{\pi^+}^2}{m_{K^{*+}}^2} = -0.287.\end{aligned}\tag{27}$$

In eqs.(26,27), the leading terms of chiral perturbation have been kept. The decay widths have been computed

$$\Gamma(K_{e3}^+) = 0.233 \times 10^{-17} GeV, \quad \Gamma(K_{e3}^0) = 0.483 \times 10^{-17} GeV.\tag{28}$$

The comparisons with experimental data are shown in Table I.

Decays of $\tau \rightarrow K^*(892)\nu$ and $\tau \rightarrow K_1(1400)\nu$

It is similar to the calculation of $\tau \rightarrow \rho\nu$ and $\tau \rightarrow a_1\nu$ [1], we obtain

$$\begin{aligned}\Gamma(\tau \rightarrow K^*(892)\nu) &= \frac{G^2}{32\pi} \sin^2\theta_c g^2 m_{K^*}^2 m_\tau^3 \left(1 - \frac{m_{K^*}^2}{m_\tau^2}\right)^2 \left(1 + 2\frac{m_{K^*}^2}{m_\tau^2}\right) = 0.326 \times 10^{-13} GeV, \\ B(\tau \rightarrow K^*(892)\nu) &= 1.46\%, \\ \Gamma(\tau \rightarrow K_1\nu) &= \frac{G^2}{32\pi} \sin^2\theta_c g^2 \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-1} \frac{m_{K^*}^4}{m_{K_1}^2} m_\tau^3 \left(1 - \frac{m_{K^*}^2}{m_\tau^2}\right)^2 \left(1 + 2\frac{m_{K^*}^2}{m_\tau^2}\right) = 0.831 \times 10^{-14} GeV, \\ B(\tau \rightarrow K^*(1400)\nu) &= 0.373\%.\end{aligned}\tag{29}$$

Decays of ϕ , $K^*(892)$, $K_1(1400)$, and $f_1(1510)$ mesons

In this theory, the vertices of $\phi K \bar{K}$, $K^* K \pi$, $K_1 K^* \pi$, $K_1 K \rho$, $K_1 K \omega$, and $f_1(1510) K^* \bar{K}$ contain even numbers of γ_5 , therefore, they are the processes with normal parity and the vertices of these processes can be derived from eq.(3). In this section the calculation of the decay widths of theses processes have been provided.

Decays of $\phi \rightarrow K\bar{K}$

In chiral limit, the vertex of this process has been found from eq.(3)

$$\mathcal{L}_{\phi K\bar{K}} = \frac{i2\sqrt{2}}{g}\phi_\mu(K^+\partial^\mu K^- + K^0\partial^\mu \bar{K}^0). \quad (30)$$

In deriving eq.(30), eq.(8) have been used. The numerical results of the decays are

$$\Gamma(\phi \rightarrow K^0\bar{K}^0) = 1.11MeV, \quad \Gamma(\phi \rightarrow K^+K^-) = 1.7MeV, \quad \frac{\Gamma(\phi \rightarrow K^0\bar{K}^0)}{\Gamma(\phi \rightarrow K^+K^-)} = 0.66. \quad (31)$$

Decays of $K^*(892) \rightarrow K\pi$

In chiral limit and using eq.(3), the vertex of this process has been obtained

$$\begin{aligned} \mathcal{L}_{K^*K\pi} = \frac{2i}{g}\{ & \sqrt{2}\pi^+(K_\mu^-\partial^\mu K^0 - K_\mu^0\partial^\mu K^-) + \sqrt{2}\pi^-(\bar{K}_\mu^0\partial^\mu K^+ - K_\mu^+\partial^\mu \bar{K}^0) \\ & + \pi^0(K_\mu^-\partial^\mu K^+ - K_\mu^+\partial^\mu K^- - \bar{K}_\mu^0\partial^\mu K^0 + K_\mu^0\partial^\mu \bar{K}^0)\} \end{aligned} \quad (32)$$

The numerical results of the decay widths are

$$\Gamma(K^* \rightarrow K^0\pi^+) = 25.4MeV, \quad \Gamma(K^* \rightarrow K^+\pi^0) = 14.0MeV, \quad \Gamma_{tot} = 39.4MeV. \quad (33)$$

Decays of $K_1(1400)$

In chiral limit the vertex of $K_1 \rightarrow K^*\pi$ has been derived from eq.(3)

$$\begin{aligned} \mathcal{L}_{K_1K^*\pi} &= f_{abi}\pi^i\{AK_{1\mu}^{*a}K^{b\mu} + Bp_\pi^\mu p_\pi^\nu K_{1\mu}^a K_\nu^{*b}\}, \\ A &= \frac{2}{f_\pi}(1 - \frac{1}{2\pi^2g^2})^{-\frac{1}{2}}(m_{K_1}^2 - m_{K^*}^2)(1 - \frac{2c}{g})(1 - \frac{3}{4\pi^2g^2}), \\ B &= -\frac{2}{f_\pi}(1 - \frac{1}{2\pi^2g^2})^{-\frac{1}{2}}\frac{1}{2\pi^2g^2}(1 - \frac{2c}{g}), \end{aligned} \quad (34)$$

where $m_{K_1}^2$ is determined by eq.(12). In the expression of A (34), eq.(12) has been applied.

The numerical results are

$$\Gamma(K_1 \rightarrow K^{*+}\pi^0) = 42.MeV, \quad \Gamma(K_1 \rightarrow K^{*0}\pi^+) = 2\Gamma(K_1 \rightarrow K^{*+}\pi^0), \quad \Gamma_{tot} = 126.MeV. \quad (35)$$

The vertex of $K_1 K \rho$ can be found from eq.(3) and it is just the formula obtained by using following formula as the A in eq.(34)

$$A = \frac{2}{f_\pi} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} \{m_{K_1}^2 - m_{K^*}^2 - (m_{K_1}^2 - m_\rho^2) \left[\frac{2c}{g} + \frac{3}{4\pi^2 g^2} \left(1 - \frac{2c}{g}\right)\right]\}. \quad (36)$$

The decay width of $K_1 \rightarrow K \rho$ has been calculated

$$\Gamma(K_1 \rightarrow K \rho) = 19.3 MeV, \quad Branch\ ratio = 11.1(1 \pm 0.075)\% \quad (37)$$

In the same way, if ignore the mass difference of ρ and ω mesons we obtain

$$\Gamma(K_1 \rightarrow K \omega) = \frac{1}{3} \Gamma(K_1 \rightarrow K \rho). \quad (38)$$

The numerical results are

$$\Gamma(K_1 \rightarrow K \omega) = 4.12 MeV, \quad Branch\ ratio = 2.4\%. \quad (39)$$

Comparing $\Gamma(K_1 \rightarrow K \rho)$ and $\Gamma(K_1 \rightarrow K \omega)$ with $\Gamma(K_1 \rightarrow K^* \pi)$, the formers are much less than the later. Except the phase space, the differences of the formulas for these three

processes are caused by the masses of ρ , ω and K^* in the amplitude A. The cancellations in A(eq.(36)) cause the smallness of $\Gamma(K_1 \rightarrow K\rho)$ and $\Gamma(K_1 \rightarrow K\omega)$.

Decay of $K_1 \rightarrow K\gamma$

Using VMD(16), following vertex has been derived from the vertex of $K_1 K \rho$

$$\mathcal{L}_{K_1 K \gamma} = -\frac{i}{2}e\left(\frac{1}{f_\rho} + \frac{1}{f_\omega} - \frac{1}{f_\phi}\right)\frac{2}{f_\pi}\left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}}\{m_{K_1}^2 - m_{K^*}^2 - m_{K_1}^2\left[\frac{2c}{g} + \frac{3}{4\pi^2 g^2}\left(1 - \frac{2c}{g}\right)\right]\}. \quad (40)$$

The numerical result is

$$\Gamma(K_1 \rightarrow K\gamma) = 440 keV. \quad (41)$$

Decays of $f_1(1510) \rightarrow K^*(892)\bar{K}$

From eq.(3), the decay amplitude has been obtained

$$\begin{aligned} &< K^+(p_1)K^{*-}(p_2)|S|f_1(p) >= \\ &-(2\pi)^4\delta^4(p - p_1 - p_2)\frac{1}{\sqrt{8m_f E_K E_{K^*}}}e_\mu^\lambda(p)e_\nu^{\lambda'*}\{Ag^{\mu\nu} + Bp_1^\mu p_2^\nu\}, \\ &A = \frac{1}{f_\pi}\left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}}(m_f^2 - m_{K^*}^2)\left(1 - \frac{2c}{g}\right)\left(1 - \frac{3}{4\pi^2 g^2}\right), \\ &B = -\frac{1}{f_\pi}\left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}}\frac{1}{2\pi^2 g^2}\left(1 - \frac{2c}{g}\right). \end{aligned} \quad (42)$$

There are four channels in this decays and the numerical results are

$$\Gamma(f_1 \rightarrow K^+ K^{*-}) = 5.48 MeV, \quad \Gamma_{tot} = 21.9 MeV. \quad (43)$$

Decays of $\eta' \rightarrow \eta\pi\pi$

In this theory, the vertex of this process contains four γ_5 . Therefore, this is a process of

normal parity and the vertex should be derived from the eq.(3). It is well known that the masses of pion and η are proportional to light quark masses[5], therefore, in chiral limit $m_\pi, m_\eta \rightarrow 0$. However, due to the $U(1)$ problem[6] $m_{\eta'}^2$ does not approach to zero in the limit of chiral symmetry. Therefore, in chiral limit only the mass of η' meson has been kept in the amplitude of $\eta' \rightarrow \eta\pi\pi$. The calculation shows that in eq.(3) only the terms at the fourth order in derivatives contribute to $\eta' \rightarrow \eta\pi\pi$. Consequently, in the amplitude of this decay there is a factor of $\frac{1}{(4\pi)^2}$. Therefore, this theory predicts that the width of this decay is very narrow. The amplitude has been found

$$\begin{aligned}
\langle \pi^0(k_1)\pi^0(k_2)\eta(p)|S|\eta'(p') \rangle &= i(2\pi)^4\delta^4(p' - p - k_1 - k_2)\frac{1}{\sqrt{16m_{\eta'}E_\eta\omega_1\omega_2}} \\
&\frac{8}{f_\pi^4}\frac{2}{(4\pi)^2}\left\{\frac{1}{2}\left(1 - \frac{2c}{g}\right)^4(q_1^4 + q_2^4 + q_3^4) + \left(1 - \frac{2c}{g}\right)\left[\frac{2c^2}{g^2} - \left(1 - \frac{2c}{g}\right)^2\right]m_{\eta'}^4\right. \\
&\left.+ \left(1 - \frac{2c}{g}\right)\left[\frac{1}{2}\left(1 - \frac{2c}{g}\right) + \frac{4c^3}{g^3}\right]q_3^2m_{\eta'}^2 + \frac{1}{2}\left(1 - \frac{2c}{g}\right)^2\left(1 - \frac{4c^2}{g^2}\right)(q_1^2 + q_2^2)m_{\eta'}^2\right\}, \quad (44)
\end{aligned}$$

where $q_1^2 = (p' - k_1)^2$, $q_2^2 = (p' - k_2)^2$, $q_3^2 = (p' - p)^2$. The contribution of quark masses to the mass of η' is about 0.376GeV, therefore, in chiral limit $m_{\eta'} = 0.582\text{GeV}$. Using this value we obtain

$$\Gamma(\eta' \rightarrow \eta\pi^+\pi^-) = 85.7\text{keV}, \quad \Gamma(\eta' \rightarrow \eta\pi^0\pi^0) = 48.6\text{keV}. \quad (45)$$

In the range of $(0.958)^2 \geq m_{\eta'}^2 \geq 0$, we obtain

$$22.1\text{keV} \leq \Gamma(\eta' \rightarrow \eta\pi^+\pi^-) \leq 145.2\text{keV}, \quad 12.5\text{keV} \leq \Gamma(\eta' \rightarrow \eta\pi^0\pi^0) \leq 82.4\text{keV}. \quad (46)$$

Eq.(46) tells that, indeed, the decay widths are always small and the data(see Table I) prefers a nonzero $m_{\eta'}$ in chiral limit. This is consistent with the study of $U(1)$ problem in $m_{\eta'}$ [6]. Therefore, phenomenologically, in lagrangian(1) a mass term of η' should be added. The study of the $U(1)$ problem could bring something new to present theory. However, this is not the task of this paper.

Decays of $K^*(892) \rightarrow K\gamma$ and $K\pi\pi$

The decays of $K^* \rightarrow K\gamma$ and $K\pi\pi$ have been studied in ref.[7] by using gauging Wess-Zumino lagrangian. As mentioned in ref.[1], the formalism obtained from this theory is the same with the one in ref.[6]. However, in this theory the couplings are universal and VMD is a result of present theory and not an input. According to VMD, the decays of $K^* \rightarrow K\gamma$ are associated with $K^* \rightarrow Kv$. Therefore, the processes of $K^* \rightarrow K\gamma$ have abnormal parity. The vertices of K^*Kv can be found from the calculation of $\frac{1}{g}K_{a\mu}^* < \bar{\psi}\lambda_a\gamma^\mu\psi >$, which is similar with $\frac{1}{g}\omega_\mu < \bar{\psi}\gamma^\mu\psi >$ in ref.[1],

$$\mathcal{L}_{K^*Kv} = -\frac{N_c}{2g^2\pi^2}\frac{2}{f_\pi}\varepsilon_{\mu\nu\alpha\beta}d_{abc}K_{a\mu}^*\partial_\nu v_\alpha^c\partial_\beta P^b, \quad (47)$$

where P^b is a pseudoscalar meson and v_α^i is a vector meson. From eq.(47) following vertices have been obtained

$$\mathcal{L}_i = -\frac{N_c}{2\pi^2g^2}\frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}K_\mu^{*+}\partial_\beta K^+\{\frac{1}{2}\partial_\nu\rho_\alpha^0 + \frac{1}{2}\partial_\nu\omega_\alpha + \frac{\sqrt{2}}{2}\partial_\nu\phi_\alpha\}. \quad (48)$$

Using VMD(eqs.(16,17)), we obtain

$$\mathcal{L}_{K^{+*}K^+\gamma} = -\frac{e}{4\pi^2g} \frac{2}{f_\pi} \varepsilon^{\mu\nu\alpha\beta} K_\mu^+ \partial_\beta K^+ \partial_\nu A_\alpha. \quad (49)$$

The the decay width has been computed

$$\Gamma(K^{+*} \rightarrow K^+\gamma) = 43.5keV. \quad (50)$$

In the same way, it has been obtained

$$\mathcal{L}_{K^{0*}K^0\gamma} = \frac{e}{2\pi^2g} \frac{2}{f_\pi} \varepsilon^{\mu\nu\alpha\beta} K_\mu^0 \partial_\beta \bar{K}^0 \partial_\nu A_\alpha, \quad (51)$$

and the decay width is

$$\Gamma(K^{0*} \rightarrow K^0\gamma) = 175.4keV. \quad (52)$$

The experimental value of the branch ratio of $K^* \rightarrow K\pi\pi$ is less than 7×10^{-4} [4]. To understand so small branch ratio is a crucial test for present theory. There are three channels

$$K^{-*} \rightarrow K^-\pi^0\pi^0, K^-\pi^+\pi^-, \bar{K}^0\pi^-\pi^0.$$

The decay $K^{-*} \rightarrow K^-\pi^0\pi^0$ consists of $K^{-*} \rightarrow K^{-*}\pi^0$ and $K^{-*} \rightarrow K^-\pi^0$. The vertices have been obtained from eqs.(47,32)

$$\begin{aligned} \mathcal{L}_{K^{-*}K^+\pi^0} &= -\frac{N_c}{\sqrt{2}\pi^2g^2} \frac{2}{f_\pi} \varepsilon^{\mu\nu\alpha\beta} K_\mu^- \partial_\nu K_\alpha^+ \partial_\beta \pi^0, \\ \mathcal{L}_{K^{-*}K^+\pi^0} &= \frac{2i}{g} K_\mu^- \partial^\mu K^+ \pi^0. \end{aligned} \quad (53)$$

These two vertices lead to following amplitude of $K^{-*} \rightarrow K^{-}\pi^0\pi^0$

$$\mathcal{M} = \frac{\sqrt{2}N_c}{\pi^2 g^3} \frac{2}{f_\pi} \varepsilon^{\mu\nu\alpha\beta} \epsilon_\mu^\lambda(p') p'_\nu k_{1\alpha} k_{2\beta} \left\{ \frac{1}{(p' - k_2)^2 - m_{K^*}^2} - \frac{1}{(p' - k_1)^2 - m_{K^*}^2} \right\}, \quad (54)$$

where p' , p , k_1 , k_2 are momentum of K^* , K , π^0 , and π^0 respectively. It can be seen that there is cancellation in eq.(54). This cancellation has been obtained in ref.[7]. The width calculated is

$$\Gamma(K^{-*} \rightarrow K^{-}\pi^0\pi^0) = 0.214 \text{keV}. \quad (55)$$

The second channel $K^{-*} \rightarrow K^{-}\pi^+\pi^-$ consists of three processes: direct coupling $K^{-*}K^+\pi^+\pi^-$ and indirect couplings: $K^{-*} \rightarrow \bar{K}^{0*}\pi^-$ and $\bar{K}^{0*} \rightarrow K^{-}\pi^+$, $K^{-*} \rightarrow K^{-}\rho^0$ and $\rho^0 \rightarrow \pi^+\pi^-$. The direct coupling has been found from $\frac{1}{g}K^{a\mu} < \bar{\psi}\lambda_a\gamma_\mu\psi >$ whose calculation is similar with the one from which the direct coupling $\omega\pi\pi\pi$ has been found in ref.[1]

$$\mathcal{L}_{K^*K\pi\pi} = \frac{1}{4\pi^2 g} \left(\frac{2}{f_\pi}\right)^3 \left(1 - \frac{6c}{g} + \frac{6c^2}{g^2}\right) \varepsilon^{\mu\nu\alpha\beta} K_\mu^a \partial_\nu P^b \partial_\alpha P^c \partial_\beta P^d d_{abe} f_{cde}, \quad (56)$$

where P stands for pseudoscalar field. Eq.(56) leads to

$$\mathcal{L}_{K^{-*}K^+\pi^+\pi^-} = -\frac{i}{2\sqrt{2}\pi^2 g} \left(\frac{2}{f_\pi}\right)^3 \left(1 - \frac{6c}{g} + \frac{6c^2}{g^2}\right) \varepsilon^{\mu\nu\alpha\beta} K_\mu^- \partial_\nu K^+ \partial_\alpha \pi^- \partial_\beta \pi^+. \quad (57)$$

From eqs.(32,47) following vertices have been obtained

$$\begin{aligned} \mathcal{L}_{K^{-*}K^0\pi^+} &= -\frac{N_c}{2\sqrt{2}\pi^2 g^2} \frac{2}{f_\pi} \varepsilon^{\mu\nu\alpha\beta} K_\mu^- \partial_\nu K_\alpha^0 \partial_\beta \pi^+, \\ \mathcal{L}_{\bar{K}^{0*}K^+\pi^-} &= \frac{2\sqrt{2}i}{g} \pi^- \bar{K}_\mu^0 \partial^\mu K^+, \end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{K^{*-}K^+\rho^0} &= -\frac{N_c}{4\pi^2g^2}\frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}K_\mu^-\partial_\beta K^+\partial_\nu\rho_\alpha^0, \\
\mathcal{L}_{\rho^0\pi\pi} &= \frac{2}{g}\epsilon_{3jk}\rho_\mu^0\pi_j\partial^\mu\pi_k,
\end{aligned} \tag{58}$$

where $\mathcal{L}_{\rho^0\pi\pi}$ is from ref.[1]. These vertices lead to the amplitude

$$\begin{aligned}
\mathcal{M}_{K^{*-}\rightarrow K^-\pi^+\pi^-} &= \frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}\epsilon_\mu(p')p'_\nu k_{+\alpha}k_{-\beta}\left\{\frac{1}{2\sqrt{2}\pi^2g}\right. \\
&\quad \left. \left(\frac{2}{f_\pi}\right)^2\left(1 - \frac{6c}{g} + \frac{6c^2}{g^2}\right) + \frac{N_c}{\pi^2g^3}\left[\frac{1}{(p' - k_-)^2 - m_{K^*}^2} - \frac{1}{(p' - p)^2 - m_\rho^2}\right]\right\},
\end{aligned} \tag{59}$$

where p' , p , k_- are momentum of η' , η , and π^- respectively. The width calculated is

$$\Gamma(K^{*-} \rightarrow K^-\pi^+\pi^-) = 1.21keV. \tag{60}$$

In the same way, the amplitude of $K^{*-} \rightarrow \bar{K}^0\pi^-\pi^0$ has been obtained

$$\begin{aligned}
\mathcal{M}_{K^{*-}\rightarrow\bar{K}^0\pi^-\pi^0} &= \frac{2}{f_\pi}\varepsilon^{\mu\nu\alpha\beta}\epsilon_\mu(p')p'_\nu k_{0\alpha}k_{-\beta}\left\{\frac{1}{2\pi^2g}\left(\frac{2}{f_\pi}\right)^2\left(1 - \frac{6c}{g} + \frac{6c^2}{g^2}\right)\right. \\
&\quad \left. - \frac{N_c}{\sqrt{2}\pi^2g^3}\left[\frac{1}{(p' - k_0)^2 - m_{K^*}^2} + \frac{1}{(p' - k_-)^2 - m_{K^*}^2}\right] + \frac{\sqrt{2}N_c}{\pi^2g^3}\frac{1}{(p' - p)^2 - m_\rho^2}\right\}.
\end{aligned} \tag{61}$$

the width calculated is

$$\Gamma(K^{*-} \rightarrow \bar{K}^0\pi^-\pi^0) = 1.23keV. \tag{62}$$

The total width is 2.65 keV which is below the limit. From eqs.(59,61) it can be seen that there are cancellations too in these two amplitudes. In these processes there are subprocesses of normal parity and abnormal parity and the relative signs between these subprocesses

have been determined without any ambiguity. Because all the vertices are revealed from the lagrangian(1). This is the universality of the couplings of this theory. Both the smallness of the phase space and the cancellations cause the smallness of the branch ration of $K^* \rightarrow K\pi\pi$.

Electromagnetic decays of mesons

In this section the processes: $\phi \rightarrow \eta\gamma$, $\eta \rightarrow \gamma\gamma$, $\eta' \rightarrow \rho\gamma, \omega\gamma$, and $\eta' \rightarrow \gamma\gamma$ have been studied. These processes have been studied in ref.[7]. The formulas obtained in this theory are the same with the ones derived from gauging Wess-Zumino lagrangian in ref.[7]. However, as mentioned above, in this theory there is universality of couplings and VMD is not an input. In the vertices of these processes the number of γ_5 is odd and they are processes of abnormal parity. In ref.[1], $\langle \bar{\psi}\gamma_5\psi \rangle$ has been evaluated(see eq.(177) of ref.[1]). In the same way $\langle \bar{\psi}\lambda_8\gamma_5\psi \rangle$ has been evaluated. We are interested in the vertices of ηvv and $\eta' vv$ which have been found as

$$\begin{aligned}
\mathcal{L}_{\eta vv} &= \frac{N_c}{(4\pi)^2} \frac{4}{g^2} \varepsilon^{\mu\nu\alpha\beta} \eta \left\{ \left(-\sqrt{\frac{2}{3}} \sin\theta + \frac{1}{\sqrt{3}} \cos\theta \right) (\partial_\mu \omega_\nu \partial_\alpha \omega_\beta + \partial_\mu \rho_\nu^i \partial_\alpha \rho_\beta^i) \right. \\
&\quad \left. - \left(\sqrt{\frac{2}{3}} \sin\theta + \frac{2}{\sqrt{3}} \cos\theta \right) \partial_\mu \phi_\nu \partial_\alpha \phi_\beta \right\}, \\
\mathcal{L}_{\eta' vv} &= \frac{N_c}{(4\pi)^2} \frac{4}{g^2} \varepsilon^{\mu\nu\alpha\beta} \eta' \left\{ \left(\sqrt{\frac{2}{3}} \cos\theta + \frac{1}{\sqrt{3}} \sin\theta \right) (\partial_\mu \omega_\nu \partial_\alpha \omega_\beta + \partial_\mu \rho_\nu^i \partial_\alpha \rho_\beta^i) \right. \\
&\quad \left. + \left(\sqrt{\frac{2}{3}} \cos\theta - \frac{2}{\sqrt{3}} \sin\theta \right) \partial_\mu \phi_\nu \partial_\alpha \phi_\beta \right\},
\end{aligned} \tag{63}$$

where θ is the mixing angle between η and η' . Combining VMD(eqs.(16,17)) and eqs.(63),

the decay widths of the physical processes have been found

$$\begin{aligned}
\Gamma(\eta \rightarrow \gamma\gamma) &= \frac{\alpha^2}{16\pi^3} \frac{m_\eta^3}{f_\eta^2} (2\sqrt{\frac{2}{3}} \sin\theta - \frac{1}{\sqrt{3}} \cos\theta)^2, \\
\Gamma(\eta' \rightarrow \gamma\gamma) &= \frac{\alpha^2}{16\pi^3} \frac{m_{\eta'}^3}{f_{\eta'}^2} (2\sqrt{\frac{2}{3}} \cos\theta + \frac{1}{\sqrt{3}} \sin\theta)^2, \\
\Gamma(\phi \rightarrow \eta\gamma) &= \frac{\alpha}{48\pi^4 g^2} \frac{m_\phi^3}{f_\eta^2} (1 - \frac{m_\eta^2}{m_\phi^2})^3 (\sqrt{\frac{2}{3}} \sin\theta + \frac{2}{\sqrt{3}} \cos\theta)^2, \\
\Gamma(\omega \rightarrow \eta\gamma) &= \frac{\alpha}{96\pi^4 g^2} \frac{m_\omega^3}{f_\eta^2} (1 - \frac{m_\eta^2}{m_\omega^2})^3 (-\sqrt{\frac{2}{3}} \sin\theta + \frac{1}{\sqrt{3}} \cos\theta)^2, \\
\Gamma(\rho \rightarrow \eta\gamma) &= \frac{3\alpha}{32\pi^4 g^2} \frac{m_\rho^3}{f_\eta^2} (1 - \frac{m_\eta^2}{m_\rho^2})^3 (-\sqrt{\frac{2}{3}} \sin\theta + \frac{1}{\sqrt{3}} \cos\theta)^2, \\
\Gamma(\eta' \rightarrow \rho\gamma) &= \frac{9\alpha}{32\pi^4 g^2} \frac{m_{\eta'}^3}{f_{\eta'}^2} (1 - \frac{m_\rho^2}{m_{\eta'}^2})^3 (\sqrt{\frac{2}{3}} \cos\theta + \frac{1}{\sqrt{3}} \sin\theta)^2, \\
\Gamma(\eta' \rightarrow \omega\gamma) &= \frac{\alpha}{32\pi^4 g^2} \frac{m_{\eta'}^3}{f_{\eta'}^2} (1 - \frac{m_\omega^2}{m_{\eta'}^2})^3 (\sqrt{\frac{2}{3}} \cos\theta + \frac{1}{\sqrt{3}} \sin\theta)^2. \tag{64}
\end{aligned}$$

There are two values for the mixing angle θ [4]. $\theta = -10^\circ$ from quadratic mass formula and $\theta = -23^\circ$ from linear mass formula. According to ref.[8], the two photon decays of η and η' favor $\theta = -20^\circ$. In this theory, $\theta = -20^\circ$ gives a better fits too. In chiral limit, we take $f_\eta = f_{\eta'} = f_\pi$. The numerical results are shown in Table I.

Conclusion

In this paper two new mass formulas have been obtained. The theoretical values of the hadronic decay rates are lower than data. The worse one is $\phi \rightarrow K\bar{K}$ which is less than data by 30%. The corrections from strange quark mass should take responsibility for these

deviations. In ref.[5] the corrections of strange quark mass to f_K and f_η have been studied. All other results agree with data well. Especially, this theory provides better understanding of the smallness of $\Gamma(K_1 \rightarrow K\rho, K\omega)$, $\Gamma(K^* \rightarrow K\pi\pi)$ and the decay of $\eta' \rightarrow \eta\pi\pi$. f_π , m_π , m_η , m_ρ , and g are not only inputs here and they are also inputs of ref.[1]. It is needed to point out that the introduction of vector and axial-vector fields to the theory is not based on gauge invariance, but on minimum coupling principle. This opens a door to introduce other mesons to the theory. In chiral limit, the cut-off determined in ref.[1] is 1.6 GeV. The mass of $f_1(1510)$ is closer to this value. However, we still obtain pretty good result of the decay $f_1(1510) \rightarrow K^*K$.

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Table 1: Table I Summary of the results

	Experimental	Theoretical
f_π	0.186GeV	input
m_π	0.138 Gev	input
m_{K^+}	0.494 Gev	input
m_{K^0}	0.498 Gev	input
m_η	0.548 Gev	input
$m_{\eta'}$	0.958 Gev	input
m_ρ	0.77GeV	input
m_{K^*}	0.892GeV	input
m_ϕ	1.02GeV	input
g		0.35 input
m_{K_1}	$1402 \pm 7 \text{ MeV}$	1.51 GeV
$m_{f_1(1510)}$	$1512 \pm 4 \text{ MeV}$	1.64 GeV
$g_{\phi\gamma}$	$0.081(1 \pm 0.05) \text{ GeV}^2$	0.086 GeV^2
$\langle r^2 \rangle_K$	$0.34 \pm 0.05 \text{ fm}^2$	0.33 fm^2
$\lambda_+(K_{l3}^+)$	0.0286 ± 0.0022	0.0239
$\xi(K_{l3}^+)$	-0.35 ± 0.15	-0.284

$\lambda_+(K_{l3}^0)$	0.03 ± 0.0016	0.0245
$\xi(K_{l3}^0)$	-0.11 ± 0.09	-0.287
$\Gamma(K_{e3}^+)$	$0.256(1 \pm 0.015) \times 10^{-17} \text{GeV}$	$0.233 \times 10^{-17} \text{GeV}$
$\Gamma(K_{e3}^0)$	$0.493(1 \pm 0.016) \times 10^{-17} \text{GeV}$	$0.483 \times 10^{-17} \text{GeV}$
$B(\tau \rightarrow K^*(892)\nu)$	$(1.45 \pm 0.18)\%$	1.46%
$\Gamma(\tau \rightarrow K_1(1400)\nu)$		0.373%
$\Gamma(\phi \rightarrow K^0 \bar{K}^0)$	$1.52(1 \pm 0.03) \text{ MeV}$	1.11MeV
$\Gamma(\phi \rightarrow K^+ K^-)$	$2.18(1 \pm 0.03)$	1.7MeV
$K^*(892) \rightarrow K\pi$	$49.8 \pm 0.8 \text{MeV}$	39.4 M3V
$\Gamma(K^{+*} \rightarrow K^+ \gamma)$	$50.3(1 \pm 0.11) \text{keV}$	43.5keV
$\Gamma(K^{0*} \rightarrow K^0 \gamma)$	$116.2(1 \pm 0.10) \text{keV}$	175.4keV
$f_1(1510) \rightarrow K^*(892) \bar{K}$	$35 \pm 15 \text{MeV}$	22.MeV
$\Gamma(K_1(1400) \rightarrow K^*(892)\pi)$	$163.6(1 \pm 0.14) \text{ MeV}$	126 MeV
$B(K_1(1400) \rightarrow K\rho)$	$(3.0 \pm 3.0)\%$	11.1%
$B(K_1(1400) \rightarrow K\omega)$	$(2.0 \pm 2.0)\%$	2.4%
$\Gamma(K_1 \rightarrow K\gamma)$		440keV
$\Gamma(\eta' \rightarrow \eta \pi^+ \pi^-)$	$87.8(\pm 0.12) \text{keV}$	85.7keV
$\Gamma(\eta' \rightarrow \eta \pi^0 \pi^0)$	$41.8(\pm 0.11) \text{keV}$	48.6keV

$\Gamma(\eta \rightarrow \gamma\gamma)$	$0.466(1 \pm 0.11)\text{keV}$	0.619keV
$\Gamma(\phi \rightarrow \eta\gamma)$	$56.7(1 \pm 0.06)\text{keV}$	91.4keV
$\Gamma(\rho \rightarrow \eta\gamma)$	$57.5(1 \pm 0.19)$	61.4keV
$\Gamma(\omega \rightarrow \eta\gamma)$	$7.0(1 \pm 0.26)\text{keV}$	7.84keV
$\Gamma(\eta' \rightarrow \gamma\gamma)$	$4.26(1 \pm 0.14)\text{keV}$	4.88keV
$\Gamma(\eta' \rightarrow \rho\gamma)$	$60.7(1 \pm 0.12)\text{keV}$	63.0keV
$\Gamma(\eta' \rightarrow \omega\gamma)$	$6.07(1 \pm 0.18)\text{keV}$	5.86keV